Chapter 1: Overview of Heat Transfer

1.1 What is Heat Transfer?

Thermal energy is related to the temperature of matter. For a given material and mass, the higher the temperature, the greater its thermal energy. Heat transfer is a study of the exchange of thermal energy through a body or between bodies which occurs when there is a temperature difference. When two bodies are at different temperatures, thermal energy transfers from the one with higher temperature to the one with lower temperature. Heat always transfers from hot to cold.

Table 1 shows the common SI and English units and conversion factors used for heat and heat transfer rates. Heat is typically given the symbol $Q$, and is expressed in joules (J) in SI units. The rate of heat transfer is measured in watts (W), equal to joules per second, and is denoted by $q$. The heat flux, or the rate of heat transfer per unit area, is measured in watts per area (W/m²), and uses $q''$ for the symbol.

<table>
<thead>
<tr>
<th>SI Units</th>
<th>English Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal Energy ($Q$)</td>
<td>1 J 9.4787 × 10⁻⁴ Btu</td>
</tr>
<tr>
<td>Heat Transfer Rate ($q$)</td>
<td>1 J/s or 1 W 3.4123 Btu/h</td>
</tr>
<tr>
<td>Heat Flux ($q''$)</td>
<td>1 W/m² 0.3171 Btu/h-ft²</td>
</tr>
</tbody>
</table>

1.2 Three Modes of Heat Transfer

There are three modes of heat transfer: conduction, convection, and radiation. Any energy exchange between bodies occurs through one of these modes or a combination of them. Conduction is the transfer of heat through solids or stationery fluids. Convection uses the movement of fluids to transfer heat. Radiation does not require a medium for transferring heat; this mode uses the electromagnetic radiation emitted by an object for exchanging heat.

1.2.1 Conduction

Conduction is at transfer through solids or stationery fluids. When you touch a hot object, the heat you feel is transferred through your skin by conduction. Two mechanisms explain how heat is transferred by conduction: lattice vibration and particle collision. Conduction through solids occurs by a combination of the two mechanisms; heat is conducted through stationery fluids primarily by molecular collisions.

In solids, atoms are bound to each other by a series of bonds, analogous to springs as shown in Figure 1.1. When there is a temperature difference in the solid, the hot side of the solid experiences more vigorous atomic movements. The vibrations are transmitted
through the springs to the cooler side of the solid. Eventually, they reach an equilibrium, where all the atoms are vibrating with the same energy.

Solids, especially metals, have free electrons, which are not bound to any particular atom and can freely move about the solid. The electrons in the hot side of the solid move faster than those on the cooler side. This scenario is shown in Figure 1.2. As the electrons undergo a series of collisions, the faster electrons give off some of their energy to the slower electrons. Eventually, through a series of random collisions, an equilibrium is reached, where the electrons are moving at the same average velocity. Conduction through electron collision is more effective than through lattice vibration; this is why metals generally are better heat conductors than ceramic materials, which do not have many free electrons.

In fluids, conduction occurs through collisions between freely moving molecules. The mechanism is identical to the electron collisions in metals.

The effectiveness by which heat is transferred through a material is measured by the thermal conductivity, $k$. A good conductor, such as copper, has a high conductivity; a poor conductor, or an insulator, has a low conductivity. Conductivity is measured in watts per meter per Kelvin (W/mK). The rate of heat transfer by conduction is given by:

$$ q_{\text{conduction}} = -kA \frac{\Delta T}{\Delta x} $$

where $A$ is the cross-sectional area through which the heat is conducting, $\Delta T$ is the temperature difference between the two surfaces separated by a distance $\Delta x$ (see Figure 1.3). In heat transfer, a positive $q$ means that heat is flowing into the body, and a
negative \( q \) represents heat leaving the body. The negative sign in Eqn. 1.1 ensures that this convention is obeyed.

### 1.2.2 Convection

Convection uses the motion of fluids to transfer heat. In a typical convective heat transfer, a hot surface heats the surrounding fluid, which is then carried away by fluid movement such as wind. The warm fluid is replaced by cooler fluid, which can draw more heat away from the surface. Since the heated fluid is constantly replaced by cooler fluid, the rate of heat transfer is enhanced.

**Natural convection** (or free convection) refers to a case where the fluid movement is created by the warm fluid itself. The density of fluid decreases as it is heated; thus, hot fluids are lighter than cool fluids. Warm fluid surrounding a hot object rises, and is replaced by cooler fluid. The result is a circulation of air above the warm surface, as shown in Figure 1.4.

**Forced convection** uses external means of producing fluid movement. Forced convection is what makes a windy, winter day feel much colder than a calm day with same temperature. The heat loss from your body is increased due to the constant replenishment of cold air by the wind. Natural wind and fans are the two most common sources of forced convection.

Convection coefficient, \( h \), is the measure of how effectively a fluid transfers heat by convection. It is measured in W/m²K, and is determined by factors such as the fluid density, viscosity, and velocity. Wind blowing at 5 mph has a lower \( h \) than wind at the same temperature blowing at 30 mph. The rate of heat transfer from a surface by convection is given by:

\[
q_{\text{convection}} = -hA \cdot (T_{\text{surface}} - T_\infty)
\]

(1.2)

where \( A \) is the surface area of the object, \( T_{\text{surface}} \) is the surface temperature, and \( T_\infty \) is the ambient or fluid temperature.
1.2.3 Radiation

Radiative heat transfer does not require a medium to pass through; thus, it is the only form of heat transfer present in vacuum. It uses electromagnetic radiation (photons), which travels at the speed of light and is emitted by any matter with temperature above 0 degrees Kelvin (-273 °C). Radiative heat transfer occurs when the emitted radiation strikes another body and is absorbed. We all experience radiative heat transfer everyday; solar radiation, absorbed by our skin, is why we feel warmer in the sun than in the shade.

The electromagnetic spectrum classifies radiation according to wavelengths of the radiation. Main types of radiation are (from short to long wavelengths): gamma rays, x-rays, ultraviolet (UV), visible light, infrared (IR), microwaves, and radio waves. Radiation with shorter wavelengths are more energetic and contains more heat. X-rays, having wavelengths ~10^{-9} m, are very energetic and can be harmful to humans, while visible light with wavelengths ~10^{-7} m contain less energy and therefore have little effect on life. A second characteristic which will become important later is that radiation with longer wavelengths generally can penetrate through thicker solids. Visible light, as we all know, is blocked by a wall. However, radio waves, having wavelengths on the order of meters, can readily pass through concrete walls.

Any body with temperature above 0 Kelvin emits radiation. The type of radiation emitted is determined largely by the temperature of the body. Most "hot" objects, from a cooking standpoint, emit infrared radiation. Hotter objects, such as the sun at ~5800 K, emits more energetic radiation including visible and UV. The visible portion is evident from the bright glare of the sun; the UV radiation causes tans and burns.

The amount of radiation emitted by an object is given by:

\[ q_{\text{emitted}} = \varepsilon \sigma T^4 \]  

(1.3)

where \( A \) is the surface area, \( T \) is the temperature of the body, \( \sigma \) is a constant called Stefan-Boltzmann constant, equal to 5.67×10^{-8} \text{ W/m}^2\text{K}^4, and \( \varepsilon \) is a material property called emissivity. The emissivity has a value between zero and 1, and is a measure of how efficiently a surface emits radiation. It is the ratio of the radiation emitted by a surface to the radiation emitted by a perfect emitter at the same temperature.

The emitted radiation strikes a second surface, where it is reflected, absorbed, or transmitted (Figure 1.5). The portion that contributes to the heating of the surface is the absorbed radiation. The percentage of the incident radiation that is absorbed is called the absorptivity, \( \alpha \). The amount of heat absorbed by the surface is given by:

\[ q_{\text{absorbed}} = \alpha I \]  

(1.4)

where \( I \) is the incident radiation. The incident radiation is determined by the amount of radiation emitted by the object and how much of the emitted radiation actually
strikes the surface. The latter is given by the *shape factor*, \( F \), which is the percentage of the emitted radiation reaching the surface. The net amount of radiation absorbed by the surface is:

\[
q_{\text{absorbed}} = F \cdot \alpha \cdot \varepsilon \cdot \sigma \cdot A \cdot T^4
\]  

(1.5)

For an object in an enclosure, the radiative exchange between the object and the wall is greatly simplified:

\[
q_{\text{enclosure}} = -\sigma \varepsilon \cdot A_{\text{object}} \cdot (T_{\text{object}}^4 - T_{\text{wall}}^4)
\]  

(1.6)

This simplification can be made because all of the radiation emitted by the object strikes the wall (\( F_{\text{object-wall}} = 1 \)).

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**Problems**

1. Define the following heat transfer situations as either conduction, convection, radiation, or a combination of the three. Please also clearly state what two objects the mode of heat transfer is between and the direction of heat transfer.

   For example: A person with a headache holds a cold ice pack to his/her forehead.

   **Answer:** Conduction occurs from the person’s forehead to the ice pack.

   a. The sun shines brightly on a car, making the black upholstery very hot.

   b. A small 4” fan is installed in the back of a computer to help cool the electronics.

   c. Potatoes are boiled in water.

   d. A turkey is being roasted in the oven.

   e. An ice cube is placed on a metal tray and left out of the freezer.

2. A loaf of freshly baked bread is left to cool on a cooling rack. The dimension of the loaf is as described in the figure, and its surface temperature is 120 °C. The temperature of the air is 20 °C, and the convection coefficient from the loaf to air is 10 W/m²K. Emissivity of the bread is 0.76, and its conductivity is 0.121.

   Based on these information, calculate the total heat loss from the bread.
3. A cup of hot coffee sits on the table. Describe all modes of heat transfer that contributes to the cooling of the coffee.

References

Chapter 2: Cooking Methods and Materials

2.1 Cooking Methods

Cooking methods can be characterized into two main types: dry heat cooking and moist heat cooking. Dry heat cooking heats foods in the absence of water, and includes methods such as baking and roasting, broiling, and pan-frying. Moist heat cooking uses water to heat food, and includes boiling, simmering, braising, and steaming.

2.2.1 Dry Heat Cooking

Ovens are used in many dry heat methods. Electric ovens use two heating coils, located at the top and bottom of the oven. The bottom coil is used for baking and roasting; the top is used for broiling. The heating coils are simply resistive elements which are heated by passing an electric current through them.

In traditional ovens, when bottom coil is heated, the air inside the oven is heated primarily by conduction and natural convection. The heat is then transferred to the food, which is heated by the natural convection current. In a convection oven, the heat transfer is enhanced by the use of a fan. The fan creates forced convection within the oven, which not only heats the food faster but also encourages even distribution of heat.

Heat transfer inside an oven is actually more complicated than simply conduction and convection. The heating element emits a considerable amount of radiation which also contributes to the heating of the food. In addition, the walls of the oven become heated as well, emitting their own radiation. Broiling takes advantage of the radiation from the coil to rapidly heat the top of the food. Broiling uses only the top coil, which suppresses the natural convection current since the hot air is blocked by the ceiling of the oven. The radiation from the top coil (which is often set to a very high temperature) heats the surface of the food to high temperatures, promoting browning.

Cooking on the stovetop utilizes conduction through a pan to heat the contents. In dry heat cooking, heat is conducted directly from the pan to the food, as shown in Figure 2.1. The heat is generated by resistive elements on electric stovetops.

![Figure 2.1 Dry heat cooking on the stovetop](image)
2.2.2 Moist Heat Cooking

Moist heat methods use water in various states to heat food. The most common state is boiling, where water is heated by conduction through a pot on stovetop, and the heat is transferred to the food through convection (Figure 2.2). Water boils at 100 °C at sea level; therefore, boiling allows the food to cook at a constant temperature of 100 °C. Although this provides a convenient, consistent cooking medium, one major disadvantage is the lack of browning and the flavors that accompany the browning reaction. Browning of food occurs at temperatures above ~150 °C, which cannot be reached by moist cooking methods. The boiling point of water can be raised slightly to reduce the cooking time. Impurities, such as salt, increase the boiling temperature by a few degrees. Pressure cookers also effectively raise the temperature of boiling water by increasing the pressure inside the pot. The relationship between pressure and boiling point is discussed in greater detail later on in the course.

![Figure 2.2 Moist heat cooking](image)

Various states of water are used for cooking, and there are special names for some of them. Poaching uses warm water just before boiling. Simmering refers to cooking in water when it has just begun to boil. Boiling uses vigorously boiling water. Steaming uses the water in vapor phase to heat the food. The temperature of steam is typically a few degrees hotter than the temperature of boiling water.

2.2.3 Other Methods

Deep frying is classified as a dry heat method, although the food is heated indirectly by convection through the oil. Oils can reach a higher temperature than water; this is why the food can be browned during frying but not in boiling. Typical frying temperatures range between 180 and 200 °C.

Microwave has become a popular cooking appliance, mainly because of its ability to heat foods quickly. It uses electromagnetic radiation to agitate water molecules, which then produce heat due to friction. You will study more about microwave ovens later on in the course.
2.2 Cooking Materials

Most cooking containers are metal or ceramic, due to their ability to withstand high temperatures. Common metals are aluminum, copper, tin, stainless steel, and cast iron. Ceramics include glass, porcelain, earthenware, and stoneware. In addition, non-stick coatings and enamel coatings may be used on the surfaces to improve the properties.

Thermal conductivity, density, and specific heat are three very important factors in determining what cooking material is best suited for the kitchen. A high conductivity material tends to have a more even temperature distribution than one with a low thermal conductivity. A material with low specific heat requires less thermal energy to heat; therefore, it heats faster than one with a high specific heat. Thermal diffusivity ($\alpha$) is the combination of the three properties:

$$ \alpha = \frac{k}{\rho \cdot c_p} $$

where $k$ is the thermal conductivity, $\rho$ is the density, and $c_p$ is the specific heat. Thermal diffusivity measures the effectiveness by which a material conducts thermal energy with respect to its ability to store thermal energy. A material with high $\alpha$ is characterized by a quick response to the changes in surrounding temperatures. A material with low $\alpha$ takes longer to reach a steady state condition, but is excellent at retaining heat once heated.

In addition to the effect of material properties, the thickness (mass) of pot also changes the heating characteristics. Thicker pots tend to allow more uniform heat distribution; however, they take longer to heat. The size and placement of the heating element with respect to the pot is also of concern.

| Table 2.1 Properties of Common Cooking Materials |
|-----------------|-----------------|-----------------|-----------------|
|                | $\rho$ (kg/m$^3$) | $k$ (W/mK) | $c_p$ (J/kgK) | $\alpha$ ($10^{-6}$ m$^2$/s) |
| Aluminum       | 2780            | 170          | 880            | 70                           |
| Cast iron      | 7870            | 70           | 450            | 21                           |
| Copper         | 8900            | 400          | 385            | 117                          |
| Stainless steel| 8000            | 15           | 480            | 3.7                          |
| Glass          | 2600            | 4            | 800            | 1.9                          |

2.2.1 Metals

Copper has the highest conductivity among all common cooking materials. Copper pans are highly treasured due to their ability to distribute heat evenly. However, copper tends to oxidize, leaving a black tarnish on its surface which also reduces its conductivity. To prevent oxidation, copper pots need constant cleaning and polishing. In addition, copper can be toxic if it diffuses into food in large amounts. Copper pans are often lined with tin to overcome these difficulties. The high cost of copper pans keep them from being used in majority of households.
Aluminum pans have the second highest thermal conductivity, next to copper, and are available at much lower cost. Aluminum is non-toxic and non-reactive, and also lightweight. Anodized aluminum has a thin coating of aluminum oxide (Al$_2$O$_3$) on the surface. Aluminum oxide forms as a result of aluminum reacting with oxygen at high temperature, and is a very hard ceramic suitable for protection from scratches.

Stainless steel is an alloy of iron, chromium, and nickel, and is desired due to its strength and resistance to corrosion. However, it is a relatively poor conductor, and often must be lined with copper or aluminum to improve the heat distribution.

Cast iron is typically iron alloyed with a small amount of carbon to increase strength. Cast iron pans are very prone to corrosion, and must be “seasoned” to prevent rusting. Seasoning a pan involves coating it with oil, then heating at moderate heat for a few hours. This fills up the small pores on the surface of the pan. Cast iron pans are generally heavy duty, allowing even heating despite the low conductivity and an ability to retain heat well.

2.2.2 Ceramics

Ceramic is usually a compound of metallic and non-metallic elements. They are poor conductors of heat, but are excellent at retaining heat once heated. They are very resistant to corrosion and are non-toxic. Due to their ability to retain heat, dishes made of thick ceramic materials will keep foods warm longer than metallic serving dishes of comparable shape and size.

A common problem with most ceramics is their tendency to crack due to thermal stress. Since they are such poor conductors of heat, there may be a large temperature difference between one side of the pan and the other. In such case, the warmer side expands more than the other, causing the pan to crack. Ceramic pans are seldom used on stovetops for this reason; ovens allow even heating from all sides, preventing large thermal gradients.

2.2.3 Coating Materials

Two common materials used for coating are Teflon (non-stick coating) and glass enamel. Teflon is a polymer which covers small roughness on pan surfaces and forms a very smooth finish, preventing food from sticking. Teflon coated pans are popular for low fat cooking; however, the coating is easily scratched by metal utensils. Glass enamel is formed by fusing glass powders to pan surfaces. The enamel improves the chemical resistance of the pan and prevents corrosion; however, they may crack when exposed to sudden changes in temperature.

Problems

1. A high quality stainless steel pot is often designed with a thick bottom, with aluminum embedded inside. Describe why these enhancements improve the overall performance of the pot.
2. Hot tea is poured into two cups of identical shape and size. One cup is made of a ceramic material, and the other is made of aluminum. Which one is likely to cool faster, and why?

References


Chapter 3: Steady-State Conduction

3.1 Steady State and Transient State

If you heat a pan on a stove, it takes a while for the pan to heat up to cooking temperature, after which the temperature of the pan remains relatively constant. The latter state is called the steady state, where there is no temporal change in temperatures. When the system is still changing with time, it is in transient state. The rate of conduction through an object at steady-state is given by:

\[ q = -kA \frac{\Delta T}{\Delta x} \]  

(3.1)

where \( k \) is the conductivity of the material, \( A \) is the cross-sectional area through which the heat is conducting, and \( \Delta T \) is the temperature difference between two surfaces separated by a distance \( \Delta x \).

3.2 One-Dimensional Conduction

One-dimensional heat transfer refers to special cases where there is only one spatial variable – the temperature varies in one direction only. A model used often to calculate the heat transfer through a 1-D system is called the thermal circuit model. This model simplifies the analysis of heat conduction through composite materials.

In this model, each layer is replaced by an equivalent resistor called the thermal resistance. An analysis much like a circuit analysis follows. For conduction, the thermal resistance is expressed as:

\[ R_{\text{conduction}} = \frac{L}{kA} \]  

(3.2)

where \( L \) is the thickness of the layer, \( k \) is the thermal conductivity of the layer, and \( A \) is the cross-sectional area. When there is more than one layer in the composite, the total resistance of the circuit must be calculated. The total resistance for layers in series is simply the sum of the resistances:

\[ R_{\text{series}} = \sum R \]  

(3.3)

For resistors in parallel, the total resistance is given by:

\[ \frac{1}{R_{\text{parallel}}} = \sum \frac{1}{R} \]  

(3.4)

The convection at the surface must also be expressed as a resistor:
\[ R_{\text{convection}} = \frac{1}{hA} \]  \hspace{1cm} (3.5)

Once the total resistance of a structure is found, the heat flow through the layers can be found by:

\[ q = \frac{T_{\text{initial}} - T_{\text{final}}}{R_{\text{total}}} \]  \hspace{1cm} (3.6)

where \( T_{\text{initial}} \) and \( T_{\text{final}} \) refers to the temperatures at the two ends of the thermal circuit (analogous to voltage difference in an electrical circuit) and \( q \) is the heat flow through the circuit (current).

**Example:** Consider a composite structure shown on the right. Conductivities of the layer are: \( k_1 = k_3 = 10 \text{ W/mK} \), \( k_2 = 16 \text{ W/mK} \), and \( k_4 = 46 \text{ W/mK} \). The convection coefficient on the right side of the composite is 30 W/m²K. Calculate the total resistance and the heat flow through the composite.

First, draw the thermal circuit for the composite. The circuit must span between the two known temperatures; that is, \( T_1 \) and \( T_\infty \).
Next, the thermal resistances corresponding to each layer are calculated:

\[ R_1 = \frac{L}{kA} = \frac{0.2}{(10)(0.1)} = 0.2 \]

Similarly, \( R_2 = 0.09 \), \( R_3 = 0.15 \), and \( R_4 = 0.36 \)

\[ R_5 = \frac{1}{hA} = \frac{1}{(30)(0.13)} = 0.26 \]

To find the total resistance, an equivalent resistance for layers 1, 2, and 3 is found first. These three layers are combined in series:

\[ R_{1,2,3} = \sum R \\
= R_1 + R_2 + R_3 \\
= 0.2 + 0.09 + 0.15 \\
= 0.44 \]

The equivalent resistor \( R_{1,2,3} \) is in parallel with \( R_4 \):

\[ \frac{1}{R_{1,2,3,4}} = \frac{1}{R_{1,2,3}} + \frac{1}{R_4} \\
= \frac{1}{0.44} + \frac{1}{0.36} \\
= 5.05 \]

\[ R_{1,2,3,4} = 5.05^{-1} \]

\[ = 0.20 \]

Finally, \( R_{1,2,3,4} \) is in series with \( R_5 \). The total resistance of the circuit is:

\[ R_{total} = R_{1,2,3,4} + R_5 = 0.46 \quad \leftarrow \text{total thermal resistance} \]

The heat transfer through the composite is:
\[ q = \frac{T_1 - T_w}{R_{\text{total}}} \]
\[ = \frac{100 - 20}{0.46} \]
\[ = 173.9 \text{ W.} \quad \leftarrow \text{heat flow through the composite} \]

Problems

1. A rectangular slab, 2 cm thick, is measured to be 100 °C on one side and 96.2 °C on the other side. The slab is 20 cm by 20 cm. Calculate the rate of heat transfer through the slab if the conductivity of the slab is 170 W/mK.

2. For a composite shown below, construct the thermal circuit model. Find the total thermal resistance of the composite. The thermal conductivities are: \( k_1 = k_3 = 80 \text{ W/mK} \), \( k_2 = 120 \text{ W/mK} \), \( k_4 = 100 \text{ W/mK} \), and \( k_5 = 150 \text{ W/mK} \).

3. For the same composite, air temperature on the left side is 20 °C, and on the right side is 80 °C. Convection coefficient is \( h = 15 \text{ W/m}^2\text{K} \) on both sides. What is the rate of heat transfer through the composite?

References

Chapter 4: Transient Conduction

4.1 Transient State

Transient state is a state of non-equilibrium, when the temperatures are still changing with time. Some simple examples are the pre-heating time of ovens, when you are waiting for the temperature inside the oven to reach a specified temperature. However, even in a heated oven, cooking occurs by transient conduction because the heat is conducted through the food as it cooks. The internal temperature of the food, therefore, is usually increasing during cooking.

4.2 Lumped Body Model

The simplest case of transient conduction is the lumped body model. This model considers the heat transfer between a solid and a surrounding fluid. The main assumption in the model is that the solid can be approximated to have a uniform temperature; that is, there are no temperature gradients within the solid. Furthermore, the surrounding fluid must be large enough so that its temperature remains a constant throughout the process. This approximation can be made when the following condition is satisfied:

$$Bi = \frac{hL_c}{k} \leq 0.1 \quad (4.1)$$

where \( h \) is the convection coefficient, \( k \) is the conductivity of the solid, and \( L_c \) is the characteristic length of the solid, defined as the volume/area ratio. For a wall of thickness \( 2L \), \( L_c = L \); for a cylinder, \( L_c = r/2 \); for a sphere, it is \( r/3 \). The number \( Bi \) is a dimensionless parameter called the Biot number.

For a lumped body, the temperature of the solid which is introduced into the fluid at time \( t = 0 \) is given by:

$$\frac{\Theta}{\Theta_i} = \frac{T(t) - T_\infty}{T_i - T_\infty} = \exp \left(-\frac{hA}{mc_p}t\right) \quad (4.2)$$

where \( A \) is the surface area of the solid, \( m \) is the mass of the solid, \( c_p \) is the specific heat of the solid, \( T_i \) is the initial temperature of the solid, and \( T_\infty \) is the temperature of the fluid. The newly introduced variable \( \Theta/\Theta_i \) is the "normalized temperature," which measures the temperature in relative terms. That is, the normalized temperature measures the percent change in temperature rather than exact change in temperature in °C. Once the normalized temperature is known, the actual temperature of the solid can be calculated based on initial temperature and the surrounding temperature.

4.3 Semi-Infinite Solid

The second approximation often applied to analyze transient conduction is the semi-infinite approximation. Semi-infinite solids can be visualized as very thick walls with
one side exposed to some fluid. The other side, since the wall is very thick, remains unaffected by the fluid temperature. This condition is expressed as \( T(\approx t) = T_i \), where \( T_i \) is the initial wall temperature. This is illustrated in Figure 4.1. The condition at the exposed side of the wall is called the \textit{boundary condition}.

4.3.1 Constant temperature boundary condition

One possible condition for the wall surface is a constant temperature (Fig. 4.1a). In this case, the temperature inside the wall at time \( t \), at distance \( x \) from the surface, is given by:

\[
\Theta = \frac{T(x,t) - T_{\text{surface}}}{T_i - T_{\text{surface}}} = \text{erf} \left( \frac{x}{2\sqrt{\alpha t}} \right)
\]

where \( T_{\text{surface}} \) is the constant wall temperature and \( \alpha \) is the \textit{thermal diffusivity} of the wall. Thermal diffusivity, described in Chapter 2, is related to the thermal conductivity, and is the measure of how quickly heat is dissipated in a material. The function \( \text{erf} \) is called the \textit{Gaussian Error Function}, and values for \( \text{erf}(x) \) are often tabulated or available in graphical form for convenience. Both a table and a graph for \( \text{erf}(x) \) are attached.

4.3.2 Surface convection boundary condition

Another possible case is to have convection off the surface of the wall (Fig. 4.1b). In this case, \( h \) is assumed to be constant at all times. The temperature in the wall at time \( t \) and distance \( x \) is:
\[
\frac{\Theta}{\Theta_i} = \frac{T(x,t) - T_{\text{initial}}}{T_{\text{ambient}} - T_{\text{initial}}} = \text{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) - \left[\exp\left(\frac{h x}{k} + \frac{h^2 \alpha}{k^2}\right)\right] \text{erfc}\left(\frac{x}{2\sqrt{\alpha t}} + \frac{h \sqrt{\alpha t}}{k}\right)
\] (4.4)

where \(\text{erfc}\) is called the Complementary Error Function, equal to \(1 - \text{erf}(x)\). The values for \(\text{erfc}(x)\) are also attached.

### 4.4 One-Dimensional Conduction

One-dimensional transient conduction refers to a case where the temperature varies temporally and in one spatial direction. For example, temperature varies with \(x\) and time. Three cases of 1-D conduction are commonly studied: conduction through a plate, in a cylinder, and in a sphere. The three geometries are shown in Figure 4.2. In all three cases, the surface of the solid is exposed to convection.

The exact analytical solutions to the three cases are very complicated. However, an approximate solution can be obtained by using graphical tools. The graphs allow you to find the centerline temperature at any given time, and the temperature at any location based on the centerline temperature. The graphs for the three geometries are attached.

#### 4.4.1 Parameters

Three parameters are needed to use the charts: Normalized centerline temperature, the Fourier Number, and the Biot Number. The definition for each parameter are listed below:

1. **Normalized centerline temperature**

   \[
   \Theta^*_i = \frac{\Theta_t}{\Theta_i} = \frac{T_0 - T}{T_0 - T_{\infty}}
   \] (4.5)

   where \(T_0\) is the centerline temperature, \(T_i\) is the initial temperature, and \(T_{\infty}\) is the ambient temperature;

2. **Fourier Number**
\[ Fo = \frac{\alpha t}{L^2} \]  

where \( \alpha \) is the thermal diffusivity, \( t \) is time, and \( L \) is the half-thickness for a plate or the radius for cylinder or sphere;

\[ iii. \text{ Biot Number} \]

\[ Bi = \frac{hL}{k} \]

where \( h \) is the convection coefficient, \( k \) is the thermal conductivity, and \( L \) is as defined above.

\textit{Note, for cylindrical and spherical cases, that the } L \text{ used in the calculation of } Bi \text{ in Equation 4.7 is not the same as } L_e \text{ used in the calculation for } Bi \text{ defined in Equation 4.1.}

4.4.2 Reading the Graphs

There are two types of charts for each geometry: the first for finding the centerline temperature, and the second for finding the temperature at any location.

To find the \textit{centerline temperature}, first calculate \( Fo \) and \( Bi^{-1} \). The first chart has \( Fo \) on the horizontal axis, \( \Theta_0^* \) on the vertical, and lines representing different values of \( Bi^{-1} \). Locate the line corresponding to the \( Bi^{-1} \) you have calculated. Next, locate the value of \( Fo \) on the x-axis, and draw a vertical line from the axis so that it intersects the \( Bi^{-1} \) line. From the intersection point, draw a horizontal line. The value at which the horizontal line crosses the y-axis is the normalized centerline temperature.

To find the temperature at any location within the solid, we now will use the second chart. The second chart has \( Bi^{-1} \) on the horizontal axis, normalized temperature on the vertical axis, and lines corresponding to normalized distance, \( x/L \) or \( r/r_0 \). Calculate the normalized distance for the point which you are interested in, and locate the corresponding line on the graph. Using the calculated value of \( Bi^{-1} \), draw a vertical line through the graph. Draw a horizontal line from the point of intersection between normalized distance curve and the vertical line. The point at which the horizontal line intersects the y-axis is the normalized temperature at that location.

4.5 Multi-dimensional Conduction

The analysis of multidimensional conduction is simplified by approximating the shapes as a combination of two or more semi-infinite or 1-D geometries. For example, a short cylinder can be constructed by intersecting a 1-D plate with a 1-D cylinder. Similarly, a rectangular box can be constructed by intersecting three 1-D plates, perpendicular to each other. In such cases, the temperature at any location and time within the solid is simply the product of the solutions corresponding to the geometries used to construct the shape. For example, in a rectangular box, \( T(x^*,y^*,z^*,t) \) — the temperature at time \( t \) and location \( x^*, y^*, z^* \) — is equal to the product of three 1-D solutions: \( T_1(x^*,t), T_2(y^*,t), \) and \( T_3(z^*,t) \).
Problems

1. An aluminum sphere with a 2 cm diameter is placed into a warm water bath at 40 °C. The initial temperature of the sphere is 20 °C. The convection coefficient between the sphere and water is 400 W/m²K. Calculate the temperature at the center of the sphere after 10 seconds. (Properties of aluminum are given in Chapter 2.)

2. You are cooking a small potato in boiling water. Approximating the potato to be a sphere with diameter of 4 cm, estimate the temperature after 2 minutes at a) the center of the potato, and b) at \( r = 1 \) cm.

\[
\begin{align*}
\rho_{\text{potato}} &= 1050 \text{ kg/m}^3 \\
c_{p,\text{potato}} &= 3.64 \text{ kJ/kgK} \\
k_{\text{potato}} &= 0.55 \text{ W/mK} \\
\alpha_{\text{potato}} &= 1.5 \times 10^{-7} \text{ m}^2/\text{s} \\
h &= 400 \text{ W/m}^2\text{K}
\end{align*}
\]

References

Gaussian Error Functions

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Figure 4A.1  Centerline temperature as a function of time for a plane wall with thickness $2L$. 

\[ F = e^{-(\frac{L}{B})} (\frac{L}{B}) \]
Figure 4A.2 Temperature distribution in a plane wall with thickness $2L$
Figure 4A.3  Centerline temperature as a function of time for an infinite cylinder with radius $r_0$. 

\[ \frac{\theta_t}{\theta_e} = F_0 \]
Figure 4A.4 Temperature distribution in an infinite cylinder with radius $r_0$. 

\[
\frac{q \theta}{L - L} = \frac{\theta}{\delta}
\]

\[\ln \left( \frac{k}{h \rho c_p} \right) = \frac{1}{\theta}
\]
Figure 4A.5  Centerline temperature as a function of time for a sphere with radius $r_0$. 
Figure 4A.6  Temperature distribution in a sphere with radius $r_0$. 

\[
\frac{\frac{8}{3}L - \theta L}{L - \frac{8}{3}L} = \frac{\theta}{\theta}
\]
Chapter 5: Convection

5.1 Overview of Convection

Heat transfer using movement of fluids is called convection. In natural convection, the flow is induced by the differences between fluid densities which result due to temperature changes. Forced convection uses externally induced flow, such as wind.

The heat transfer rate for convection is given by the following equation:

\[ q = -hA(T_{\text{surface}} - T_\infty) \]  

(5.1)

where \( h \) is the convection coefficient, \( A \) is the surface area, and \( T_{\text{surface}} \) and \( T_\infty \) are the surface and ambient temperatures, respectively. The convection coefficient is a measure of how effective a fluid is at carrying heat to and away from the surface. It is dependent on factors such as the fluid density, velocity, and viscosity. Generally, fluids with higher velocity and/or higher density have greater \( h \).

5.2 Natural Convection

Density of fluid changes with temperature. In general, fluids expand as the temperature rises, and thus the density decreases (density is the mass per unit volume). Warm fluids therefore are more buoyant than cooler fluids. A hot object will heat the surrounding fluid, which rises due to the buoyancy force. The warm fluid is then replaced by cool (unheated) fluids. Similarly, cool objects will draw heat away from the surrounding fluid, which then fall due to the increased density. The cool fluid is then replaced by the warm fluid, initiating convective currents.

For a hot horizontal plate surrounded by air, convection currents form when the air above the plate start to rise, as shown in Figure 5.1. The air below the plate, however, cannot rise because the plate is blocking the flow. The heated fluid under the plate will eventually escape through the sides of the plate; however, the convective flow below the plate is very small compared to the flow on top. In general, natural convection is more pronounced (has a higher \( h \)) on the topside of a hot plate or the bottom side of a cold plate.

The convection coefficient for natural convection in gas is generally between 1 W/m²K and 20 W/m²K; typical values for liquids fall between 100 W/m²K and 1000 W/m²K.

5.3 Forced Convection

Fluid flow caused by a fan or any other external forces create forced convection. Forced convection is generally more efficient than natural convection due to the faster velocity of the currents. In forced convection, buoyancy
has little effect on the direction of flow.

The relation between flow velocity, direction, and temperature is illustrated in Figure 5.2 for a hot, horizontal surface under forced convection. Since the buoyancy does not affect the flow, the bottom side of the plate will have the same patterns. Close to the surface, the flow velocity is inevitably slowed down due to friction. Right at the surface, the velocity is actually zero. This region of retarded flow is called the boundary layer. The region of warm airflow is generally well within the velocity boundary layer, and is called the thermal boundary layer.

The convection coefficient for forced convection in gasses generally range between 50 W/m²K and 250 W/m²K. For liquids, values start around 100 W/m²K, and can be as high as 10,000 W/m²K.

5.4 Convection in Ovens

Conventional ovens use natural convection to heat foods while baking. Ovens typically contain two heating elements, on top and bottom of the oven. During baking, the bottom element heats up, which heats the air inside the oven. The hot air rises and creates a current, which helps to distribute heat throughout the oven. Natural convection currents are easily blocked by large pans, creating non-uniform temperatures within the oven.

Convection ovens improve temperature distribution by using a fan, located inside the oven, to create forced convection currents. The forced convection currents efficiently mix the air inside an oven, creating uniform temperatures even in the presence of large pans. Furthermore, the increased airflow results in a higher convection coefficient, which reduces cooking time.

Problems

1. A pot of water, 20 cm in diameter and 15 cm tall, is kept at boil on a stove. The convection coefficient on top and side surfaces is 25 W/m²K, and the room temperature is 22 °C. Calculate the heat loss due to convection, neglecting the bottom surface.

References

Chapter 6: Radiation

6.1 Introduction to Radiation

Any matter with temperature above absolute zero (0 K) emits electromagnetic radiation. In a simplified picture, radiation comes from the constantly changing electromagnetic fields of the oscillating atoms. Electromagnetic radiation can be visualized as waves traveling at the speed of light. The two prominent characters of the wave are the wavelength ($\lambda$) and frequency ($\nu$). The wavelength is the distance between crest to crest on the wave. The frequency is related to wavelength by the following:

$$\nu = \frac{c}{\lambda}$$ (6.1)

where $c$ is the speed of light, approximately equal to $3\times10^8$ m/s in vacuum. The wavelength is measured in units of length, and the frequency is given in cycles per second (hertz, Hz).

The amount of radiation emitted by a body depends on its temperature, and is proportional to $T^4$. This relation shows that as the temperature of the object increases, the amount of radiation emitted increases very rapidly. The emitted radiation will travel at the speed of light until it is absorbed by another body. The absorbing medium can be gas, liquid, or solid. Radiation does not require a medium to pass through. This is demonstrated by solar radiation which pass through interplanetary space to reach the earth.

6.2 Electromagnetic Spectrum

Electromagnetic radiation is categorized into types by their wavelengths. The types of radiation and the respective wavelength ranges are shown in Figure 6.1. Radiation with shorter wavelengths are more energetic, evident by the harmful gamma and x-rays on the shorter end of the spectrum. Radio waves, which are used to carry radio and TV signals, are much less energetic; however, they can pass through walls with no difficulty due to their long wavelengths.

The type of radiation emitted by a body depends on its temperature. In general, the hotter the object is, the shorter the wavelengths of emitted radiation, and the greater the amount. Most of the radiation we see from a cooking standpoint is in the...
infrared range (a 500 K body, for example, emits the most radiation at ~2×10^{-5} m). A much hotter body, such as the sun (~5800 K), emits the most radiation in the visible range.

The total energy emitted by a body, regardless of the wavelengths, is given by:

\[ q_{\text{emitted}} = \varepsilon \sigma \cdot A T^4 \]  

(6.2)

where \( \varepsilon \) is the emissivity of the body, \( A \) is the surface area, \( T \) is the temperature, and \( \sigma \) is the Stefan-Boltzmann constant, equal to 5.67×10^{-8} \text{ W/m}^2\text{K}^4. Emissivity is a material property, ranging from 0 to 1, which measures how much energy a surface can emit with respect to an ideal emitter (\( \varepsilon = 1 \)) at the same temperature.

### 5.3 Radiative Properties

When radiation strikes a surface, a portion of it is reflected, and the rest enters the surface. Of the portion that enters the surface, some are absorbed by the material, and the remaining radiation is transmitted through. This is shown in Figure 6.2.

The ratio of reflected energy to the incident energy is called reflectivity, \( \rho \). Similarly, transmissivity (\( \tau \)) and absorptivity (\( \alpha \)) are defined as the fraction of the incident energy that is transmitted through or absorbed by the object, respectively. The three radiative properties all have values between zero and 1. Furthermore, since the reflected, transmitted, and absorbed radiation must add up to equal the incident energy, the following can be said about the three properties:

\[ \alpha + \rho + \tau = 1 \]  

(6.3)

Generally, an object with high reflectivity has low absorptivity and emissivity. Those with low reflectivities tend to have high absorptivities and emissivities.

### 5.4 Radiative Heat Transfer

Consider the heat transfer between two surfaces, as shown in Figure 6.3. What is the rate of heat transfer into Surface \( B \)? To find this, we will first look at the emission from \( A \) to \( B \). Surface \( A \) emits radiation as described in Eqn. 6.2:

\[ q_{A,\text{emitted}} = \varepsilon_A \sigma \cdot A_A T_A^4 \]  

(6.4)

This radiation is emitted in all directions, and only a fraction of it will actually strike Surface \( B \). This fraction
is called the shape factor, $F$. The amount of radiation striking Surface $B$ is therefore:

$$q_{B, \text{incident}} = F_{A \rightarrow B} \varepsilon_A \sigma \cdot A_A T_A^4$$  \hfill (6.5)

The only portion of the incident radiation contributing to heating Surface $B$ is the absorbed portion, given by the absorptivity $\alpha_B$:

$$q_{B, \text{absorbed}} = \alpha_B F_{A \rightarrow B} \varepsilon_A \sigma \cdot A_A T_A^4$$  \hfill (6.6)

Equation 6.6 is the amount of radiation going into Surface $B$ from Surface $A$. To find the net heat transfer rate at $B$, we must now subtract the amount of radiation emitted by $B$:

$$q_{B, \text{emitted}} = \varepsilon_B \sigma \cdot A_B T_B^4$$  \hfill (6.7)

The net radiative heat transfer rate at Surface $B$ is Eqn. 6.6 minus Eqn. 6.7:

$$q_B = \alpha_B F_{A \rightarrow B} \varepsilon_A \sigma \cdot A_A T_A^4 - \varepsilon_B \sigma \cdot A_B T_B^4$$  \hfill (6.8)

### 5.5 Shape Factors

Shape factor, $F$, is a geometrical factor which is determined by the shapes and relative locations of two surfaces. Figure 6.4 illustrates this for a simple case of cylindrical source and planar surface. Both the cylinder and the plate are infinite in length. In this case, it is easy to see that the shape factor is reduced as the distance between the source and plane increases. The shape factor for this simple geometry is simply the cone angle ($\theta$) divided by $2\pi$.

Shape factors for other simple geometries are available in heat transfer texts. However, for more complicated geometries, the following two rules must be applied to find shape factors based on simple geometries.

The first is the summation rule. This rule says that the shape factor from a surface (1) to another (2) can be expressed as a sum of the shape factors from (1) to (2a), and (1) to (2b). Using this rule allows you to break up complicated geometry into smaller pieces for which the individual shape factors can be found.

The second rule is the reciprocity rule, which relates the shape factors from (1) to (2) and that from (2) to (1) as follows:

$$A_1 F_{1 \rightarrow 2} = A_2 F_{2 \rightarrow 1}$$  \hfill (6.9)

Thus, if the shape factor from (1) to (2) is known, then the...
shape factor from (2) to (1) can be found by:

\[ F_{2\to1} = \frac{A_1}{A_2} F_{1\to2} \]  

(6.10)

Problems

1. Dark baking pans often cooks foods faster than shiny ones because they absorb more heat. Determine the amount of heat transfer from the coil to the pan for an anodized aluminum and stainless steel pan. You may neglect convection and conduction, as well as radiation emitted by the pan. Radius of coil is 3mm.

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References

Chapter 7: Microwaves

7.1 Introduction

It was discussed in Chapter 6 that the electromagnetic spectrum consists of various types of radiation, characterized by wavelength ($\lambda$) and frequency ($\nu$). The frequency of the wave can be visualized as the number of wave crests that move by an observer in a second (Figure 7.1). Microwave radiation refers to the region of the spectrum with frequencies between $\sim 10^9$ Hz to $\sim 10^{11}$ Hz. This type of radiation lies between infrared radiation and radio waves.

Recall that waves with shorter wavelength (lower frequency) have higher energy. The frequency of visible (green) light at $\lambda=500$ nm is approximately $6 \times 10^{14}$ Hz. The frequency of radiation used in microwave ovens is approximately $2.5 \times 10^9$ Hz, with a wavelength of 12 cm. This means that the microwave radiation used to heat food is actually less energetic than visible light. Why, then, are microwave ovens so efficient at heating food?

7.2 Water Molecules and Microwaves

Water molecules (H$_2$O) are polar; that is, the electric charges on the molecules are not symmetric (Figure 7.2). The alignment and the charges on the atoms are such that the hydrogen side of the molecule has a positive (+) charge, and the oxygen side has a negative (-) charge.

Electromagnetic radiation have electric charge as well; the "wave" representation shown in Figure 7.1 is actually the electric charge on the wave as it flips between positive and negative. For a microwave
oscillating at $2.5 \times 10^9$ Hz, the charge changes signs nearly 5 billion times a second (2 times $2.5 \times 10^9$ Hz).

Electric charges are similar to magnetic charges: opposites attract. When oscillating electric charge from radiation interacts with a water molecule, it causes the molecule to flip (Figure 7.3). Microwave radiation used in ovens are specially tuned to the natural frequency of water molecules to maximize this interaction. Therefore, as a result of the radiation hitting the molecules, the water molecule flips back and forth 5 billion times a second. This rigorous motion of the molecules cause friction which heats the food.

7.3 Inside a Microwave Oven

Microwave radiation is produced by a device called the magnetron. The magnetron consists of four major components: an anode block, a cathode filament, a pair of permanent magnets, and an antenna.

The anode block is a hollow cylinder with fins coming out to the inside. The top view is shown in Figure 7.4. The fins are called the anode vanes, and the spaces between them are the resonant cavities.

The cathode filament is a cylindrical rod located at the center of the anode block. It serves as the source for electrons during the emission of microwave radiation.

Two permanent magnets are located at the top and bottom of the anode block, as shown in the side view in Figure 7.5. These magnets create a magnetic field inside the anode block that are parallel to the cathode filament.

An antenna is positioned so that one end goes into one of the resonant
cavities in the magnetron. The other end is located in the waveguide, which transfers the microwave radiation to the cooking chamber, much like how fiber optics is used to transfer light to remote locations.

The events which lead to the production of microwave radiation begins when an electron is emitted by the cathode filament. Since the filament and the electron both have negative charge, the electron accelerates outward, toward the anode vanes (Figure 7.6). The magnetic field due to the permanent magnets around the anode block are pointing out from the paper (represented by +).

Moving electric field from the electrons induce a magnetic field around itself. If you were traveling with the electron, facing forward, the induced magnetic field appears clockwise. On Figure 7.6b, this is denoted by the (+) above and (−) below the electron. The positive and negative magnetic field below the electron cancels each other, while the positive magnetic fields above the electron builds up to create a net force downward, perpendicular to the electric force. This net force pushes the electrons clockwise around the cathode filament.

When electrons are continuously emitted all around the cathode tube, they collectively move clockwise around the cathode filament, forming a cloud (Figure 7.7). When an electron approaches an anode vane, a positive charge is induced within the vane as electrons in the vane are repelled by the approaching free electron. Meanwhile, negative charge is induced in the neighboring anode vanes due to the accumulation of the repelled electrons. The negatively charged vanes then repel electrons rotating within the electron cloud. The effect of this is that the electron cloud forms into a pinwheel shape, as shown in
Figure 7.8. As the cloud rotates, the anode vanes closest to the "spokes" of the pinwheel has an induced positive charge, and the neighboring vanes are negatively charged. As the cloud rotates, the positive and negative charges in the vanes rapidly oscillate. The oscillation of charges create an alternating current within the resonant cavities of the anode block. This current is carried by the antenna, and released as microwave radiation inside the waveguide.

The final step in a microwave oven is to release the radiation within the cooking chamber. When the radiation exits the waveguide, it is often reflected by a rotating fan blade to evenly distribute the radiation throughout the cooking chamber (Figure 7.9). Once entering the chamber, the radiation is reflected by the chamber walls until it is absorbed by the food.

7.4 Characteristics of a Microwave

Microwave heats food by agitating the water molecules. Most foods we consume contain over 70% water by weight, making this an effective way for heating foods. However, the negative side is that food with low water contents take longer to heat in a microwave. Furthermore, frozen foods take longer to heat because the water molecules are not moved as much as in liquid water. One must also remember, when using a microwave, that foods heated in a microwave cannot become hotter than the boiling point of water, or 100°C. This is why foods cannot be browned in a microwave, and pies reheated in a microwave do not have a crisp crust like a freshly baked pie would have.

One often hears the statement, "microwaves cook foods from inside." This is because foods heated in a microwave tend to heat most rapidly in the center, and cooks much faster than what we intuitively believe from conduction of heat from outside. The statement, however, is not entirely true; microwave radiation simply has the ability to penetrate several inches into the food. Radiation with longer wavelengths tend to penetrate deep into matter, evident by the ease by which radio waves go through concrete walls. Since microwaves penetrate into food several inches down, it can heat foods quicker than if it were being heated through conduction by boiling water on the outer surface. If the food were larger than several inches, however, the middle of the food still needs to be heated by conduction.
One of the major drawbacks of microwave is the hot spots. Microwave radiation, as it reflects around the cooking chamber, interact with other reflected radiation in such way that hot and cold spots are formed inside the microwave. The phenomenon responsible for this is the interference of waves, shown in Figure 7.10. Waves overlapping in such way that the crests match one another interfere constructively, forming a hot spot. Waves which interfere destructively result in a cold spot. The uneven heating caused by these effects can be reduced significantly by using a rotary platform, included in many of the newer models of microwave ovens.

Problems

1. A typical direction for microwavable frozen food reads “Heat on high for 2 minutes, then let stand for an additional minute.” Why is the second step necessary?
Chapter 8: Thermodynamics of Water

8.1 Three States of Water

Under normal conditions, water exists in one of three states: solid, liquid, and gas. Water in its solid state is what we call ice. Pure water at sea level freezes at 0°C. Liquid water is most often used in cooking, and the temperature varies between 0°C and 100°C. Water boils at 100 °C (at sea level) and becomes vapor. The vapor state is more commonly called steam. The transition between two states is called a phase transformation.

If you take a cup of water and heat it under atmospheric conditions, the heating curve will look like one shown in Figure 8.1. In the figure, the horizontal axis shows time (or more energy added), and the vertical axis is temperature. Notice that the temperature of water stays constant with added energy during phase transformation. This is because all of the energy is used to transform one state to another that no energy is available for heating the water.

8.2 Boiling

Boiling water is one of the most commonly used heat source in cooking. Boiling water is undergoing liquid-to-gas transition, and because of this it stays at a constant temperature of approximately 100°C. This provides a convenient standard for us to control the cooking process.

As liquid water is heated, the molecules become increasingly mobile. Some of the molecules acquire enough energy to escape as vapor. These molecules exert a force on the atmosphere, called the vapor pressure. The vapor pressure is opposed by another force, created by a column of air pushing down on the pan. This pressure is the atmospheric pressure. The two pressures are illustrated in Figure 8.2. Water begins to boil when the vapor pressure overcomes the atmospheric pressure. This means that the majority of water molecules have become energetic enough to escape the surface.

The temperature at which water boils is related to the vapor pressure required for boiling, which is equal to the atmospheric
pressure. The implication of this is that as the atmospheric pressure changes, the boiling point of water changes as well. When you go up a mountain, the air pressure is lower (the column of air pushing down is smaller). Therefore, water boils at a lower temperature, and food takes longer to cook. For every 1000 ft. in altitude, the boiling point of water decreases by about 1 °C.

A clever appliance designed to take advantage of the pressure-boiling point relation is the pressure cooker. A pressure cooker is a tightly sealed pot which uses the steam from water to increase the internal pressure of the pot. As the pressure inside the pot increases, the boiling point rises, and a higher temperature can be achieved for cooking. The internal temperature can be determined by controlling the vapor pressure inside the pot. Since the internal temperature can be raised substantially above 100°C, food cooks faster.

To understand the relation between pressure and temperature, consider the following relation:

\[ PV = nRT \]  

where \( P \), \( V \), and \( T \) are the pressure, volume, and temperature of the gas, respectively, and \( n \) is the number of moles of the gas per unit volume. \( R \) is a material-specific constant, expressed in units J/mol-K. The above relation is called the ideal gas equation, and shows the following relations for any constant amount of gas:

1) at constant pressure, the volume of gas increases with increasing temperature;

2) at constant volume, the temperature of the gas increases with increasing pressure.

The boiling point of water can also be changed by adding impurities in the water. Impurities include salt, sugar, and other dissolving molecules. Generally, impurities increase the boiling point of water. A simple explanation of this is that the impurities dilute the concentration of water (the number of water molecules per unit volume decreases), and the number of molecules that can vaporize at any given temperature decreases. The result is that a higher temperature is required to achieve the same vapor pressure.

### 8.3 Freezing

Water freezes when the molecules have slowed down enough to develop bonds upon collision. The rate at which freezing occurs is governed by nucleation and growth.

Nucleation is the formation of small solids in a liquid. The clusters of solids are called the nuclei. The rate at which new nuclei form (number of nuclei per second) is the nucleation rate. Once the nuclei have formed, they become the landing sites for other molecules to attach onto. The growth rate is the rate at which the radius of a nucleus grows after formation. The solidification rate is determined by the combination of nucleation and growth rates.
The size of crystals formed during solidification is determined by the nucleation/growth processes. A solidification process with fast nucleation rate and/or slow growth rate will result in many small crystals forming. Larger crystals form from slow nucleation rate.

Most liquids decrease in volume upon solidification. Water, however, has a rather unique property of expanding during liquid-to-solid transformation. This property comes from the hexagonal structure of ice crystals; water molecules form a hexagonal crystal structure, which actually takes up more volume than if the molecules were freely slipping past one another. Consequently, ice cubes float in water.

The freezing point of water at sea level is 0°C. This temperature can be changed, however, by adding impurities in water. Sprinkling salt on road surfaces on an icy day melts the ice by lowering the melting temperature. Salt is also used in simple ice cream machines during cooling of the cream. Another consequence of the decrease in freezing point due to impurities is the soft texture of ice creams. As ice cream freezes, the remaining liquid becomes more and more concentrated with sugar and other impurities. The concentrated liquid has a much lower freezing temperature than water. As a result, ice cream never completely freezes, and retains the characteristic soft texture.

### 8.4 The Refrigeration Cycle

Refrigerators and freezers use refrigeration cycle to draw heat away from the compartments to keep foods cool. The cycle has four major components: evaporator, compressor, condenser, and an expansion valve, as shown in Figure 8.3. A working fluid, typically a refrigerant such as chlorofluorocarbons (CFC’s), flows through the cycle in a closed loop. At point A, the fluid is at a low-pressure liquid state. As it enters the evaporator, which is located inside the cooling compartment, it expands into its vapor state. The phase change is accompanied by absorption of heat from the refrigeration compartment, cooling the refrigerator. An ideal temperature for a refrigerator is ~2 °C, or just above freezing. For freezers, it is typically ~ -15 °C.

After all the working fluid has turned into vapor in the evaporator, the gaseous fluid then enters the compressor. The compressor takes the gas and pumps it to the condenser. The pressure of the fluid increases during this process. The condenser is a series of tubes that

![Figure 8.3 Refrigeration Cycle](image-url)
you typically see behind the refrigerator. As the fluid passes through these tubes, it cools by releasing heat into the surrounding air and condenses to a liquid state. The heat released by the fluid is what you feel behind the refrigerator. The heat, which was taken from the refrigeration compartment, is now released into the room.

Finally, an expansion valve is used to carry the condensed fluid back to the evaporator. The tube has a very small diameter compared to the condenser, because the tube only has to carry a small amount of liquid refrigerant as opposed to a less dense gaseous fluid.

Refrigeration keeps foods from spoiling by reducing the growth of bacteria. The cool temperature slows down the metabolism of bacteria in foods. This, however, is not the only reason why refrigerators have become such integral part of our lives. Some foods, such as fruits, taste better when they are cold. Furthermore, some foods need to be cooled to develop certain texture. An example of such case is Jell-O, which needs to be refrigerated in order to solidify.

An inevitable result of keeping the food at a low temperature is the tendency to dry out if left uncovered. Cold air cannot support as much moisture as warm air, as evident from the dry atmosphere in the winter. The air inside the refrigerator is much drier than in the room. Foods with high water contents must be protected from the dry air using covered containers or plastic wraps. Vegetable and fruit compartments in the refrigerator help keep the foods’ moisture within the drawer to prevent drying.

Freezing allows foods to be kept even longer than refrigerating by slowing the bacteria even further. In addition, freezing tend to lock in the water molecules in the form of ice, preventing foods from drying. After time, however, fluid loss will occur through sublimation (phase transition from ice to vapor), causing freezer burns. Another effect of freezing is the loss of texture in many vegetables and fruits. Plants have a high water content, much of which are trapped within cells. Water expands upon freezing and ruptures the cell walls, leading to the loss of structure and texture in the foods.

References